PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Non-classical light 20 years later: an assessment of the voyage into Hilbert space

H. J. Kimble

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Non-classical light 20 years later: an assessment of the voyage into Hilbert space

By H. J. Kimble

Norman Bridge Laboratory of Physics 12–33, California Institute of Technology, Pasadena, CA 91125, USA

Although diverse manifestations of the quantum or non-classical character of the electromagnetic field have arisen over the past two decades in quantum optics, almost without exception these observations have been made in a domain of weak coupling for which dissipation is dominant over the coherent dynamical processes associated with single quanta. By contrast, research in the area of cavity quantum electrodynamics has achieved the exceptional circumstance of strong coupling for the interaction of individual atoms with the quantized field of a high-quality resonator. The research programs in the Quantum Optics Group at Caltech attempt to explore quantum dynamical processes in this domain of strong coupling, and include investigations of photon antibunching due to quantum-state reduction, of the implementation of quantum logic with single photons, of new avenues for quantum-state synthesis and of atomic centre-of-mass motion for single atoms falling one-by-one through the field of a high-finesse optical cavity.

1. Introduction

Given that the meeting of The Royal Society on 'Highlights in Quantum Optics' took place only a few months shy of 20 years since the observation of non-classical light in the fluorescence from single atoms, I could not resist the temptation to begin the discussion by recalling those measurements made in the laboratory of Professor Leonard Mandel at the University of Rochester in June, 1977 (Kimble *et al.* 1977). Although manifestly quantum or non-classical fields such as those first observed in Mandel's group had been anticipated theoretically since the earliest days of quantum optics (Sudarshan 1963; Glauber 1965), many years passed before the 'tools of the trade' advanced to a degree sufficient to the challenge at hand, both in terms of analytical progress in identifying suitable physical systems as well as technical developments to make the experiments possible (e.g. the invention of the tunable dye laser). Following the work on photon antibunching, Mandel & Short (1983) reported measurements of sub-Possoinian photon statistics for the fluorescence from single atoms, while Saleh & Teich made similar observations with many atoms and thereby set the stage for a whole class of new experiments which 'transfer' the statistics of an external pump to the electromagnetic field (Teich & Saleh 1988; Golubev & Sokolov 1984; Yamamoto *et al.* 1986).

Another very successful player in the story of the generation of non-classical light has been parametric down conversion, as was pioneered once again by Mandel's

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group. The basic process is described by the interaction Hamiltonian

$$H_{\mathrm{I}} = \hbar\chi^{(2)} E_{\mathrm{c}} E_a^{\dagger}(\Omega) E_b^{\dagger}(-\Omega) + \mathrm{H.c.}, \tag{1.1}$$

where $\chi^{(2)}$ is the second-order susceptibility which couples the 'pump' field E_{c} to the down-converted 'signal' and 'idler' fields $(E_a^{\dagger}(\Omega), E_b^{\dagger}(-\Omega))$ of frequencies $\frac{1}{2}\omega_{\mathrm{c}} \pm \Omega$. Because $\chi^{(2)}$ is 'small' in the sense that a single pump photon in the nonlinear medium has negligible probability of generating a signal and idler pair (which in qualitative terms is a statement of weak coupling), it is possible to replace equation (1.1) by the much simpler form

$$H_{\mathrm{I}} \approx \hbar\kappa E_a^{\dagger}(\Omega) E_b^{\dagger}(-\Omega) + \mathrm{H.c.}. \tag{1.2}$$

Here, $\kappa = \langle \chi^{(2)} E_{\mathrm{c}} \rangle$ is an 'effective' coupling coefficient, where fortunately (for the moment) the complexity of the full three mode problem has been avoided. With an initial vacuum state for the signal and idler fields and for short interaction times such that $\xi = |\kappa t| \ll 1$, equation (1.2) then leads to the well-known result for the generated output state

$$|\Psi\rangle \sim (1-\xi^2)^{1/2} |0\rangle_{\Omega} |0\rangle_{-\Omega} + \xi |1\rangle_{\Omega} |1\rangle_{-\Omega}. \tag{1.3}$$

By triggering on photoelectric detection of an idler photon at frequency $-\Omega$, Hong & Mandel (1986) realized a localized one-photon state for the signal field at Ω. The fields generated in parametric down conversion have since become something of a work-horse over the past decade and have led to a wealth of beautiful non-classical phenomena, including observations of non-local interference (Ou *et al.* 1990) and violations of Bell inequalities at unprecedented levels of statistical significance (for a review, see Mandel & Wolf 1995).

It is perhaps worth noting in passing that many of these experiments are often (and incorrectly) described in terms not of the state $|\Psi\rangle$ above, but rather in terms of a 'real' two-photon state

$$|\Phi\rangle \sim |1\rangle_{\Omega} |1\rangle_{-\Omega}. \tag{1.4}$$

Such a perspective is justified only as a *post diction* (i.e. only for the set of events that produces a pair of individual 'clicks' in the signal and idler detectors). The actual state of the field is in fact $|\Psi\rangle$ (or, in fact, a suitably generalized broad-band version thereof). A source that actually produces a pair of photons (with a predetermined moment of birth) as represented by the state $|\Phi\rangle$ is yet to be realized. This state of affairs is part and parcel of the deal that was struck in moving from the full Hamiltonian of equation (1.1) to the domain of weak coupling in equation (1.2).

Yet a third non-classical field of some significance in recent years has been the so-called squeezed states of light (Kimble & Walls 1987; Giacobino & Fabre 1992). It seems to be often overlooked that precisely the same process responsible for 'pairs of photons' as in equation (1.2) is employed for the generation of squeezed light, however now in the limit of large ξ. At least for the generation of a squeezed-vacuum state, what had been the state $|\Psi\rangle$ consisting of the vacuum plus a small probability of a pair of photons now becomes a sum containing appreciable probability for multiple pairs of photons. Rather than detection by way of photon counting, the method of choice becomes homodyne detection to interrogate the quadrature amplitudes of the field. Hence on the one hand, parametric down conversion as associated with the state $|\Psi\rangle$ is said to produce a pair of photons (which is, in fact, the case only *a posteriori*), while on the other hand, squeezed states are most often described in terms of the amplitude of the field. So which is it to be then: fields or photons? Since in either case the relevant transformation is for the *amplitude of the field* (whether as in the Schrödinger wave function in equation (1.3) or in the corresponding Heisenberg field operator) and since quantum electrodynamics is afterall a quantum *field* theory, this rhetorical question has an all too obvious answer. From my perspective, altogether too many 'paradoxes' in modern quantum optics arise by adopting a description in terms of propagating 'bundles of energy', whereas the underlying theory is one of *field amplitudes* and hence shares a remarkably large overlap with classical Maxwell's theory. Hence, much of what is often called 'quantum' behaviour is nothing of the sort, having an exact correspondence in the classical theory.

As the zoology of non-classical fields expands, and in view of the preceeding discussion, one might ask to what extent this whole business is simply one of pedantics and semantics. Such matters notwithstanding, there is a fundamental significance and an operational consequence associated with manifestly quantum or non-classical fields for which the quantum fluctuations are below the vacuum-state limit (more quantitatively, for which the Glauber–Sudarshan phase-space function is singular and non-positive). Specificially, such fields must necessarily be employed in order to surpass the standard quantum limits (SQL) on precision measurment. Indeed, for the historical paradigms that illustrate the (standard) limits to quantum measurement, the limiting factor on sensitiviy can always be traced to the fluctuations of the vacuum field (including the Heisenberg microscope), as was pointed out in the pioneering work of Caves (1981). Within this context, squeezed light has found gainful employment, having been utilized to make the first measurements with sensitivity beyond the SQL (aka the vacuum or shot-noise limit) (Kimble 1992), beginning with interferometry (Xiao *et al.* 1987; Grangier *et al.* 1987) and extending to atomic spectroscopy (Polzik *et al.* 1992).

Granting then that non-classical states are of some interest, we might pause to inquire further about the nature of the physical processes that generate such states. Such a discussion leads inevitiably to the dynamics of open quantum systems. Here we note that such systems can be characterized quite generally in terms of the relationship of the internal rate χ for reversible, coherent interactions (e.g. the rate $\chi^{(2)}$ for processes as in equation (1.1)) and of the external rate Γ for dissipative loss into the environment, with the ratio $m \sim (\Gamma/\chi)^2$ serving as a critical parameter for the system's dynamics. The historical emphasis has been in a domain of weak coupling for which $m \gg 1$ and for which therefore one quantum more or less is of no consequence for the system's dynamics. This is the regime relevant to all the examples of the preceeding discussion, which is not to say that it is a domain devoid of interest, giving rise, for example, to the original paradox of Einstein, Podolsky and Rosen (Ou *et al.* 1992) and to the modern tests of Bell inequalities. However, it is a domain of very definite limitations (as, for example, in the distinction of the states $|\Psi\rangle$ and $|\Phi\rangle$ above). In terms of the allegory adopted for this presentation, operation in a domain of weak coupling restricts our voyage into Hilbert space to one of sailing always within sight of land.

By contrast, to sail beyond the known horizon to the most exotic (and general) destinations in the Hilbert space of light requires capabilities in a regime of strong coupling for which $m \ll 1$. Here, single quanta can profoundly affect the system's evolution, and the time scale $1/\chi$ associated with internal quantum dynamics cannot be scaled away. It is a non-trivial undertaking to achieve strong coupling in physics, with only a handful of demonstrations having been presented to date. Rather than attempt to offer a survey of this work, I would like instead to expand upon the concept of strong coupling with reference to a specific example, namely that of cavity quantum electrodynamics (cavity QED).

2. Cavity quantum electrodynamics

Historically, much of the work in cavity QED has arisen within the context of atomic physics with an emphasis on the alteration of atomic line positions and linewidths due to the presence of metallic or dielectric boundaries. Perhaps the premier example in this regard is the pioneering work of Drexhage (1974) who demonstrated both enhanced and inhibited spontaneous decay for molecules in the proximity of a plane boundary. Much of modern research in cavity QED continues this tradition (Hinds 1990). However, in qualitative terms many aspects of the phenomenology in this domain can be understood from the perspective of classical antenna theory with the radiating atomic dipole (or a higher-order multipoles) as the basic dynamical system which is perturbed by a modified external environment (Drexhage 1974; Dowling *et al.* 1991).

By contrast, our focus has been on a non-perturbative regime which elevates the cavity field to the level of an equal partner with the atom in the dynamics of the composite atom–cavity system. Towards this end, the target of our own work has been to achieve conditions of strong coupling for which the time scale for coherent, reversible evolution is comparable to that for irreversible decay by way of atomic spontaneous emission and of loss through the cavity mirrors. Stated in terms of the preceding discussion, if the coupling frequency of a single atom to the cavity mode is g (i.e. $2g$ is the one-photon Rabi frequency), then our experiments have achieved strong coupling with $g > (\gamma, \kappa)$, where γ is the atomic decay rate to modes other than the cavity mode and κ is the decay rate of the cavity mode itself. In this circumstance, the number of photons required to saturate an intracavity atom is $n_0 \sim \gamma^2/g^2 < 1$ and the number of atoms required to have an appreciable effect on the intracavity field is $N_0 \sim \kappa\gamma/g^2 < 1$. Note that n_0 gives the 'saturation' photon number for the atom–cavity field interaction, while N_0 serves as a measure of the 'critical' atomic number (N_0^{-1} gives the cooperativity parameter per atom) (Lugiato 1984; Carmichael 1993). In qualitative terms, n_0 and N_0 specify the role of a single photon and of a single atom, respectively.

Passing from this general discussion to a more quantitative one, we introduce a model system, which is taken to describe a single two-state atom located in a cavity formed by two spherical mirrors. The Hamiltonian H_s for this system is well known and takes the form of a sum of atomic, field and interaction terms (Jaynes & Cummings 1963),

$$\hat{H}_\mathrm{s} = \tfrac{1}{2}\hbar\omega_\mathrm{A}\hat{\sigma}^z + \hbar\omega_\mathrm{C}\hat{a}^\dagger\hat{a} + \mathrm{i}\hbar[g(\boldsymbol{r})\hat{a}^\dagger\hat{\sigma}^- - g^*(\boldsymbol{r})\hat{a}\hat{\sigma}^+]. \tag{2.1}$$

The operators $\hat{a}$ and $\hat{a}^\dagger$ are the annihilation and creation operators for the single mode of the resonator under consideration, while $\hat{\sigma}^z$ and $\hat{\sigma}^\pm$ are the Pauli operators for the atomic inversion, raising and lowering, respectively. ω_A and ω_C are the atomic and cavity resonance frequencies. The coherent coupling between the atom at position $\boldsymbol{r}$ and the cavity mode is $g(\boldsymbol{r})$, with

$$g(\boldsymbol{r}) = \left(\frac{\mu^2\omega_\mathrm{C}}{2\hbar\epsilon_0 V_m}\right)^{1/2} U(\boldsymbol{r}) \equiv g_0 U(\boldsymbol{r}), \tag{2.2}$$

where the cavity-mode function $U(\boldsymbol{r})$ is chosen so that the cavity-mode volume

$V_m = \int d^3x|U(\boldsymbol{r})|^2$. μ is the transition-dipole moment for the (assumed) two-state atom. Of course, $\hat{H}_s$ is responsible for coherent (reversible) evolution in our problem.

Rather than concentrate on the range of dynamical processes associated with $\hat{H}_s$ alone, our interest is to include the omnipresent dissipation that arises from atomic and cavity decay. There is a rather well-worn pathway in quantum optics for incorporating these decay processes which involves the derivation of a quantum master equation for the density operator $\hat{\rho}$. To the extent that the atom is weakly coupled to a continuous background of modes excluding the resonant cavity mode, a standard weak-coupling approximation can be made to connect the atom irreversibly to these external modes, with $\gamma_\parallel$ and $\gamma_\perp$ as the resulting rates for longitudinal and transverse decay to these background modes. Damping of the cavity mode through the boundaries of the resonator is accounted for by a rate κ.

By taking $\gamma \equiv \gamma_\perp = \frac{1}{2}\gamma_\parallel$ as appropriate to radiatively broadened decay to external background modes, we arrive at the following quantitative expressions for the previously introduced parameters n_0 and N_0 (Kimble 1994):

$$n_0 \equiv \left(\frac{\gamma_\perp\gamma_\parallel}{4g_0^2}\right) b = \frac{4}{3}\frac{\gamma^2}{g_0^2}, \quad N_0 \equiv \frac{2\gamma_\perp\kappa}{g_0^2} = \frac{2\gamma\kappa}{g_0^2}, \tag{2.3}$$

where $b = \frac{8}{3}$ for an average over a Gaussian standing-wave mode, while $b = 1$ for an atom with $U(\boldsymbol{r}) = 1$.

In correspondence with the discussion of the Introduction, the usual situation in quantum optics is to operate in a regime of weak coupling for which $g_0 \ll (\gamma, \kappa)$, so that $(n_0, N_0) \gg 1$, and one photon or one atom more or less is of no consequence. For example, note that a typical laser is operated with a threshold photon number $\sqrt{n_0} \sim 10^3 - 10^4$ and that an optical parametric oscillator has $\sqrt{n_0} \sim 10^4 - 10^5$ intracavity subharmonic photons at threshold, with the number of intracavity atoms at threshold $N_0 \gg 1$ in each case. In this limit of weak coupling, the quantum master equation can be solved by a system-size expansion based upon the small parameters n_0^{-1} and N_0^{-1} (Kimble 1992; Carmichael 1993), with the generic result that dynamical processes take the form of more or less classical trajectories with small bits of quantum noise. Note that in this case, the internal time scale g_0^{-1} for coherent quantum dynamics is scaled away. By contrast, in the regime of strong coupling, the internal clock which specifies coherent quantum time g_0^{-1} runs faster than the external dissipative clock $(\gamma^{-1}, \kappa^{-1})$. The atom–cavity system then has time to couple itself coherently and at least the possibility of a life of manifestly quantum dynamics before the grim reaper of dissipation enters.

These lofty goals of principle must, of course, confront the reality of current technological capability. It is a considerable challenge to achieve conditions for which $g_0 > (\gamma, \kappa)$ or equivalently, for which $(n_0, N_0) < 1$. An overview of our progress over the years is presented in figure 1, which gives the critical photon number as defined in equation (2.3) versus year for a series of experiments first at the University of Texas at Austin and more recently at Caltech. The data for this plot are described in Kimble (1994), with the exception of the last two points which are from the work of Mabuchi *et al.* (1996) and Hood *et al.* † (1997, unpublished work), respectively.

From our earliest measurements with finesse $\mathcal{F} \sim 10^2$ in the early 1980s, we have progressed to systems with $\mathcal{F} \geqslant 10^5$. Note that we have most recently implemented

† The parameters given are for our newest system in a cavity of length 10 μm and finesse 2×10^5, where we have recently observed individual atom transits as in Mabuchi *et al.* (1996).

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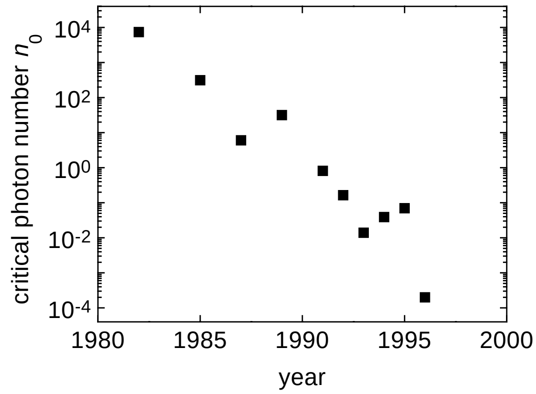

Figure 1. Critical photon number n_0 versus year.

Table 1. *Table of parameters realized in cavity QED experiments with strong coupling*

group	$\omega/2\pi$	$g_0/2\pi$	$\kappa/2\pi$	$\gamma_{\parallel}/2\pi$	g_0T_0	$n_0 \sim \beta^2/g^2$
Walther[a]	21.5 GHz	7 kHz	1.9 Hz	500 Hz	1.3π	0.04
Haroche[b]	51.1 GHz	21 kHz	0.5 kHz	5 Hz	0.9π	0.078
Kimble[c]	353 THz	7.2 MHz	0.6 MHz	5 MHz	4.3π	0.16
[d]	353 THz	11 MHz	3.5 MHz	5 MHz	2000π	0.026
[e]	353 THz	120 MHz	35 MHz	5 MHz	$10\,000\pi$	0.00023
Feld[f]	379 THz	0.34 MHz	94 kHz	50 kHz	0.2π	1.6

[a]Raithel *et al.* 1995,
[b]Brune *et al.* 1996,
[c]Kimble *et al.* 1995,
[d]Mabuchi *et al.* 1996,
[e]Hood *et al.* 1997,
[f]An *et al.* 1994, Childs *et al.* 1996.

a system with a cavity of length 10 µm and mirrors with radius $R = 10$ cm, for which (with the atom localized at an antinode)

$$(g_0, \gamma, \kappa)/2\pi = (120, 2.5, 35) \quad \text{and} \quad (n_0, N_0) = (2 \times 10^{-4}, 1.2 \times 10^{-2}). \tag{2.4}$$

The technical achievement that makes this research possible is the attainment of very high finesse for spherical mirror interferometers. Together with R. Lalezari at Research Electrooptics, we have observed $\mathcal{F} = 1.9 \times 10^6$, corresponding to mirror reflectivity $R = 0.9999984$ and to a cavity $Q = 1.8 \times 10^{10}$ (for length $l = 4$ mm) (Rempe *et al.* 1992), which remains the highest finesse on record for an optical cavity. This high finesse allows us to go to shorter cavities (and hence larger g_0 values) without unacceptable increases of the cavity decay rate κ.

Within the context of other experimental work in cavity QED, there are two experiments in the microwave domain and two in the optical domain that have achieved strong coupling for a single atom ($g_0 > (\kappa, \gamma)$) (see articles in Berman 1994). The relevant rates for these experiments as well as for two of our recent experiments with cold atoms are as listed in table 1.

For our experiments carried out in the optical domain, the transit time is large compared to the dissipative time scale ($T > 1/(\gamma, \kappa)$), and the relevant parameters become n_0, N_0 as introduced in equation (2.3). Here, the perspective is that of the characteristics of the atom–cavity system in the presence of dissipation, with non-linearity arising from the interplay of reversible coherent evolution and irreversible decay into the environment (via a master equation for the reduced degrees of freedom of the system). Since the number of photons required to saturate an intracavity atom scales as n_0, while the number of atoms required to have an appreciable effect on the intracavity field goes as N_0, single photons and individual atoms can profoundly alter the system's dynamics in the domain of strong coupling with $(n_0, N_0) < 1$, which is most certainly not the case for systems heretofore studied.

By contrast with the other entries in the table for which an atom transits the cavity in time $T < 1/(\gamma, \kappa)$, a relevant quantity is $2g_0T$, which specifies the Rabi nutation angle for an atom initially in the ground state travelling through the resonator in the presence of (initially) a single intracavity photon. By noting that the excited state population of the atom $\propto \sin^2(\theta)$, where $\theta = 2\sqrt{n}g_0T$, we can define a critical photon number analogous to our previous expression for n_0. Since $\theta_0 = \frac{1}{2}\pi$ represents a change from ground to excited state, we take for the corresponding 'critical' photon number

$$n_0 = \pi^2/16(g_0T)^2. \tag{2.5}$$

Because the microwave experiments have the ability to select the atomic velocity over an appreciable range, there is, in fact, a whole set of values of g_0T accessable, with the values quoted in the table only meant to represent some 'typical' value. Note that for all of the experiments with atomic beams, $g_0T \sim \pi$.

In either the stationary atom case or that of an atom transiting in a short time, we are thus led to introduce a 'critical' photon number n_0 as

$$n_0 \sim \beta^2/g_0^2, \tag{2.6}$$

where for $T > 1/(\gamma, \kappa)$, $\beta \sim \gamma$, while for $T < 1/(\gamma, \kappa)$, $\beta \sim 1/T$ (i.e. $\beta \sim \max[\gamma, 1/T]$). Again, note that the criterion for strong coupling is that $n_0 < 1$, and that in either the transient $[T < 1/(\gamma, \kappa)]$ or steady state regimes $[T > 1/(\gamma, \kappa)]$, extremely large effects can be wrought by individual quanta.

Of course, no two parameters such as n_0, N_0 can serve to classify the diverse phenomena that are being investigated in the realm of cavity QED with strong coupling. However, these quantities do provide an intuitively useful metric for determing the ability to explore the Hilbert space in a much more powerful fashion than is possible in a domain of weak coupling. For example, in qualitative terms, n_0 specifies a fractional precision (in units of the energy of the rms vacuum field) for measurement sensitivity beyond the SQL. The emphasis of our work as well as that of Professor Haroche and Professor Walther described in these Proceedings is to push well into the domain $(n_0, N_0) \ll 1$.

Given the preceding overview of the context and broad objectives of our research programme, we turn in the following sections to describe some of the specific activities that have been pursued in recent years, as well as to a brief discussion of research avenues that are currently being undertaken.

3. A quantum phase gate (QPG) in cavity QED

An important accomplishment that built upon our previous realization of a 'one-dimensional' atom (Turchette *et al.* 1995*a*) has been the demonstration of conditional dynamics at the single-photon level suitable for the implementation of quantum logic (Turchette *et al.* 1995*b*). Our measurements utilized the circular birefringence of an atom strongly coupled to the field of a high finesse optical cavity to rotate the polarization state of a linearly polarized 'probe' beam (i.e. a one-atom waveplate). Because the rotation angle of the probe beam can be controlled by the intensity of a circularly polarized 'pump' beam for intracavity fields with average photon number much less than one, our observations demonstrated conditional dynamics between pump and probe fields at the level of single quanta. More quantitatively, by introducing a simple but general model for the dynamics of our system, we extracted from our data conditional phase shifts $\Delta \simeq 18^\circ$ photon^{-1}. Note that beyond the context of quanutm logic and computation, the parameter Δ has model independent significance as the strength of the dispersive nonlinear interaction between intracavity fields quoted in degrees per unit of stored energy. Its large value represents a unique achievement within the field of nonlinear optics.

To explore further the prospects for quantum logic based on these capabilities, the 'truth table' for our quantum-phase gate was experimentally verified, with the measurements indicating that the transformation affected by the atom–cavity system is 'non-trivial' in that it could serve as a universal element for quantum computation. Here the quantum carriers of information (the 'qubits') are fields which propagate in two frequency offset (and hence functionally distinct) channels, with the internal state in each case specified by the circular polarization states $\sigma_\pm$. Although we have not made measurements of entanglement for the fields emerging from the QPG, detailed calculations carried out by S. Tan (University of Auckland) during a four month sabbatical stay with our group indicate that it should be possible to observe nearly maximal violations of Bell inequalities with the output fields for coherent-state inputs. Note that we had previously observed non-classical light in measurements of photon antibunching in cavity QED, which results as a dynamical consequence of quantum-state reduction in a dissipative setting (Rempe *et al.* 1991).

4. Cold atoms and cavity QED

Another significant experimental result arising from the attainment of strong coupling has been the real-time detection of single atoms transiting through a high-finesse optical cavity (Mabuchi *et al.* 1996). For this experiment, caesium atoms are dropped from a magneto-optical trap located 7 mm above a Fabry–Perot cavity, as illustrated in figure 2. By recording the reduction of the cavity transmission as an atom enters the cavity mode, we can monitor with high signal-to-noise ratio the 'trajectory' of an individual atom as it transits through the cavity, with an example of our data given in figure 2. Indeed, there is preliminary evidence that the motion of single atoms through the standing-wave structure of the cavity field is being observed. If this interpretation is supported by subsequent measurements, then the resolution of these initial observations begins to approach the standard quantum limit for sensing the atom position (within a factor of roughly 4). More generally, table 1 gives some sense of the advance that this work represents relative to other experiments in the area of cavity QED, where the product of coherent coupling constant g with the transit time T_0 is seen to be $g_0T_0 \sim 2\pi \times 10^3$, whereas for all other experiments (which employ atomic beams), $g_0T_0 \sim \pi$. Beyond the domain of cavity QED, this work represents an improvement in detection time of roughly 10^5 over previous work

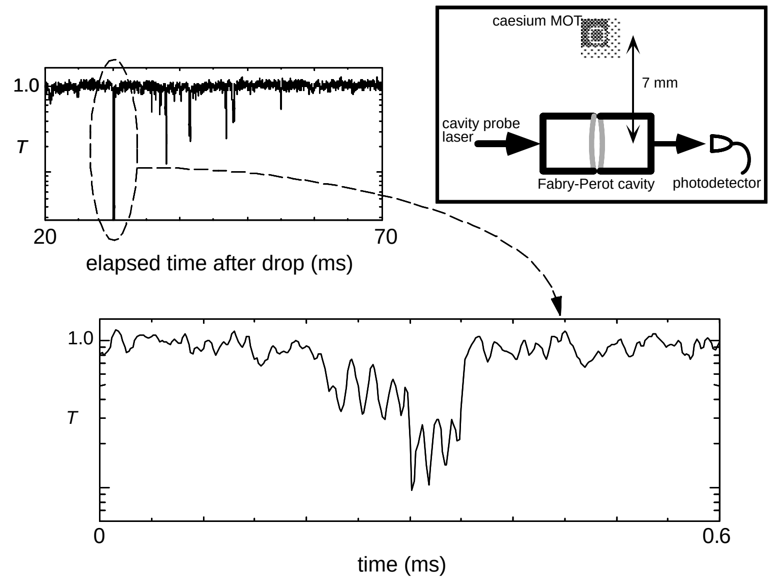

Figure 2. Real-time atom detection signals, shown on two different timescales. The transmission T is given in units of intracavity photon number. Inset provides a schematic diagram of the experimental setup.

aimed at detection of single atoms or molecules by absorption (Wineland *et al.* 1987; Moerner & Kador 1989).

To progress past these initial measurements, we are working to localize an atom within the mode of an optical cavity by employing the quantized versions of the usual semi-classical light forces. For our experiments the energy $\hbar g$ associated with the coherent coupling of atom and cavity for a single intracavity photon can be much larger than the thermal energy E_{k} of laser cooled atoms. In more concrete terms, note that a new system (Hood *et al.* 1997) has achieved $T_g/T_0 \approx 750$, with $T_g = 2\hbar g_0/k_{\mathrm{B}}$ corresponding to the splitting of the first excited state of the Jaynes–Cummings ladder and $T_0 \sim 15\ \mu\mathrm{K}$ being the temperature of the atomic sample in our initial attempts with polarization-gradient cooling (there is, in addition, a velocity $v = \sqrt{2ah}$ from the earth's acceleration a over a distance $h \sim 5$–10 mm, which we are working to compensate). Hence, as a slow atom moves within the cavity field, the spatial dependence of the coupling energy $\hbar g(\boldsymbol{r})$ can lead to substantial forces that modify (indeed, confine) the atomic motion. Further note that as the atom travels through the cavity mode-function $g(\boldsymbol{r}) \equiv g_0 U(\boldsymbol{r})$ with a constant external drive $\mathcal{E}$, the intracavity field x is likewise modified and is determined in a self-consistent fashion from the atomic position $\boldsymbol{r}$. For example, for weak excitation of the cavity, $x \sim (\mathcal{E}/\kappa)/[1+2C_1|U(\boldsymbol{r})|^2]$, where $C_1 = 1/N_0$. With the parameters of Mabuchi *et al.* (1996), this means that the motion of an atom from a node to an antinode can cause a change of the intracavity intensity by a factor $x^2 > 10^2$, with a correspondingly large modification of the atom's motion itself. Taken in concert, the self-consistent evolution of cavity field and atomic motion can be utilized to localize the atom within the cavity. Apart from the novelty of trapping an atom with a fraction of a photon, such an atom can then be exploited for a variety of fundamental experiments.

The basic plan that we are pursuing involves a magneto-optical trap (MOT) situated a few millimeters above a high finesse optical cavity (figure 2). When the MOT is switched off, the atoms fall between the cavity mirrors with some small fraction of the atoms actually transiting through the cavity. Since we are now able to monitor in real time the transit of individual atoms through the cavity, the next step is to switch the intracavity field to trap the atom. Here we make use of the fact that the lower peak of the vacuum-Rabi splitting corresponds to an attractive potential, with the average 'well depth' increasing with increasing probability for occupation of the lower dressed state, at least in the limit of intracavity fields with photon number $n \leqslant 1$. Thus the strategy is first to monitor for the presence of an atom with an intracavity field of photon number $n_1 \ll 1$ which would have a small effect on the atomic centre-of-mass motion. Having detected an atom (as in figure 2), we would then switch an external drive tuned to the lower peak of the vacuum-Rabi spectrum to produce an intracavity photon number $n_2 \leqslant 1$ and a corresponding population in the lower dressed-state manifold, thereby creating a confining potential sufficient to trap the atom. The sensibility of this basic strategy is supported by the results of numerical simulations carried out by A. S. Parkins, A. C. Doherty and collaborators in the group of Professor D. F. Walls at the University of Auckland.

5. New avenues for the synthesis of field states in cavity QED

Together with Professor P. Zoller's group, we have developed a new idea for the synthesis of quantum field states of the form $|\phi\rangle = \sum_m c_m |m\rangle_{\mathrm{F}}$, where $|m\rangle_{\mathrm{F}}$ are Fock states for a single-mode field and the c_m can be chosen experimentally with flexibility and broad lattitude (Parkins *et al.* 1993, 1995). The basic idea involves a mapping of internal Zeeman coherences from atom to cavity field via adiabatic passage as a single, suitably prepared atom transits through the cavity. Note that our analysis includes both atomic and cavity damping, with atomic decay playing no significant role since, for a broad range of conditions, the atom remains in a dark state throughout its transit.

Our more recent work in this area has taken two directions. The first is to incorporate a second quantized cavity mode into the analysis; the particular situation that we consider is that of two degenerate cavity modes of orthogonal polarizations. In this case, W. Lange has shown how to extend our original treatment to produce field states of entangled polarization, and has devised and analysed a new (realistic) scheme for the generation of so-called GHZ states of light (where GHZ denotes Greenberger–Horne–Zeilinger) (Lange & Kimble 1997). The second direction that we have taken is an attempt to adapt the ideas from the adiabatic passage scheme to the situation of an atom localized within the cavity mode, as in our work with cold atoms described above. Since it is very difficult to produce an atom 'on demand' to make a fast transit through the cavity field (transit times much less than 100 ns are required for the realization of the scheme analysed in Parkins *et al.* 1993, 1995), we plan to use instead a cold atom that is more or less stationary within the cavity mode. Turning the coupling to the quantized field 'on' and 'off' will be accomplished via lambda-type transitions to and from an uncoupled (non-interacting) ground state (Law & Eberly 1996). For example, for the case of caesium, the $6S_{1/2}$, $F = 4 \Leftrightarrow 6P_{3/2}$, $F' = 4$ transition would be near resonance (with detuning $\varDelta$) with the quantized cavity mode (with coupling g_0), while the $6S_{1/2}$, $F = 3 \Leftrightarrow 6P_{3/2}$, $F' = 4$ would be far from resonance and driven by a 'classical' field of Rabi frequency $\varOmega$. In a Raman scheme, the effective coupling then becomes $g_{\mathrm{eff}}(t) = \varOmega(t) g_0/\varDelta$ and hence can be varied in time via control of $\varOmega(t)$. Following this general theme, C. K. Law (from the Rochester Theory Center for Optics) and I have shown how to generate a

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

deterministic bit stream of photon pulses with a 'user' controlled pulse shape (Law & Kimble 1997).

6. Evanescent fields of whispering gallery modes for cavity QED

We have carried out an extensive analytical and numerical treatment of the bound state structure and dynamics for an atom trap formed from the whispering gallery modes (WGMs) of a dielectric microsphere (Vernooy & Kimble 1997*a*). The coupling of the quantized internal and external atomic degrees of freedom is found to play a fundamental role in the quantum dynamics of the resulting *atomic gallery*. In particular, the radiative processes for a cold atom near a microsphere are modified due to the special symmetry of the atom gallery, the WGM mode structure, and the finite extent of the centre-of-mass (CM) wavepacket. While it is well known that radiative processes are fundamentally modified for an atom outside of a dielectric sphere, previous calculations (Jhe & Kim 1995; Chew 1987; Klimov & Letokhov 1996) have not included the quantum mechanical nature of the CM state. Given the paucity of fully quantum calculations for realistic *three-dimensional* configurations, our work represents an important step forward in the understanding of the role of CM wavepackets in cavity QED.

In terms of experimental progress, we are exploring the whispering gallery modes of small fused-silica spheres (diameter $\sim 100\,\mu\mathrm{m}$) (Braginsky *et al.* 1989; Mabuchi & Kimble 1994; Knight *et al.* 1996*a*,*b*). Although there are a number of complex issues related to mode identification and coupling, a central question relates to the quality factors Q that can be attained with these resonators. Projected values range to $Q \simeq 10^{11}$ (Braginsky *et al.* 1989) (which would correspond to a cavity storage time of $50\,\mu\mathrm{s}$ at 852 nm), with $Q \simeq 0.8 \times 10^{10}$ recently reported (Gorodetsky *et al.* 1995). In collaboration with V. Ilchenko from Moscow State University, D. Vernooy in our group has recently achieved $Q \simeq 8 \times 10^9$ for excitation at 670, 780 and 850 nm.

7. Future research directions

Beyond the research described in the preceding sections, we are also pursuing the following research activities.

(1) Investigations of continuous quantum measurement at or beyond the standard quantum limit, including quantum dynamical processes leading to entanglement of internal and external degrees of freedom (Holland *et al.* 1991; Storey *et al.* 1992*a*,*b*; Averbukh *et al.* 1994; Herkommer *et al.* 1996). Here the context of the research is that of the dynamics of continuously monitored quantum systems whereby the strong coupling of atom and cavity implies a back reaction of one sub-system on the other as a result of a measurement (Caves & Milburn 1987; Gagen *et al.* 1993; Milburn 1996). As applied to measurements of the atomic CM motion, we are particularly interested in the ultimate limits with which the atomic trajectory can be followed.

(2) Investigations of bound states and wavepacket dynamics in cavity QED (Vernooy & Kimble 1997*b*). As an atom becomes yet 'colder' and better localized within the cavity mode, it becomes necessary to consider the full, non-perturbative wavepacket dynamics including bound states for the system. We have thus undertaken an investigation of structure and dynamics for an atom strongly coupled to a cavity mode in the domain for which $E_\mathrm{k} < \hbar g$. Beginning with the spectrum of eigenvalues, we have extended the familiar dressed states for the Jaynes–Cummings

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Hamiltonian to include bound CM states that arise either because of the intrinsic spatial variation of $g(\boldsymbol{r})$ or because of an externally applied atomic potential $V_{\text{ext}}(\boldsymbol{r})$, as for example in an RF Paul trap (Monroe *et al.* 1995). Spatially localized eigenstates for both the external motion in a potential well and for the internal atom–field interaction are termed 'well-dressed' states. Our analysis explores the interplay of the finite spatial extent of a CM wavepacket $\psi(\boldsymbol{r})$ with the quantum field mode structure $g(\boldsymbol{r})$. Implicit in the eigenvalues of the well-dressed states are new CM-dependent spatial and temporal scales that can lead to novel 'collapses' and 'revivals' of internal atom–field coherence, as well as to modifications of spontaneous emission.

(3) Operation of a laser with 'one-and-the-same' atom. In the usual domain of weak coupling, the threshold for laser operation is characterized by a cooperativity parameter $D = M/M_0$, where M specifies the atomic inversion (in number of atoms) and M_0 is defined in a fashion analogous to N_0, but depends upon the particular details of the level scheme and pumping process (Bonifacio & Lugiato 1982). For conventional lasers, $(M_0, n_0) \gg 1$, and consequently a large number of atoms and photons are associated with the lasing threshold, which occurs for $D \simeq 1$. By contrast, note that in our experiments we have already achieved $N_0 \ll 1$, so that it seems reasonable to project 'lasing' for $N \sim 1$ atom, which is a projection substantiated in general terms by the work of Mu & Savage (1992) for one-atom lasers. Recall also that $n_0 \ll 1$ photon, so that the 'laser' would operate with one (and the same) atom and much less than one photon. Following earlier work by Professor C. Savage and Professor P. Zoller, we are currently investigating the $6D_{5/2}$, $F = 6 \rightarrow 6P_{3/2}$, $F' = 5$ transition in atomic caesium in collaboration with Dr C. K. Law. Pumping is to be by way of a two-photon transition from the $6S_{1/2}$, $F = 4$ ground state to the $6D_{5/2}$ level, thus forming a Raman-type laser (with a two-photon initial stage) (Ritsch & Zoller 1992; Ritsch *et al.* 1992).

(4) Explorations of the evolution of open quantum systems in the presence of feedback. As intimated in various components of the previous discussions, we intend to investigate quantum-limited feedback for the atom–cavity system, following the theoretical lead of G. Milburn and colleagues (Wiseman & Milburn 1993*a*,*b*; Slosser & Milburn 1995). As noted by Wiseman (1993*a*), the general master equation for homodyne-mediated feedback shares much in common with that describing continuous quantum measurement of position, so that our work in this area is formally, as well as practically, a natural extension to pursue. Moreover, as applied to studies of atomic CM motion, the setting of cavity QED with a single intracavity atom is unique with respect to the bandwidth and efficiency with which quantum-limited information about atomic motion can be extracted. Hideo Mabuchi is leading our group's research efforts in this field, both in terms of developing an understanding of the 'sensor' (quantum parameter estimation, as in Mabuchi (1996)) and of the 'actuator' (coherent control of quantum dynamics).

The research described herein has been carried out in the Quantum Optics Group at the California Institute of Technology. The graduate students responsible for the progress described herein are D. Bass, J. Buck, N. Georgiades, C. Hood, H. Mabuchi, T. Lynn, Q. Turchette and D. Vernooy. Senior members of the group include Dr M. Chapman, Dr A. Furusawa (Nikon Advanced Research Labs) and Dr W. Lange (now at the MPQ in Garching). We have benefited greatly from interactions with and extended visits by members of Professor D. F. Walls's group at the University of Auckland, including Dr S. Parkins and Dr S. Tan, as well as Mr A. Doherty. Dr C. K. Law was a visiting scholar from the University of Rochester during the autumn, 1996. The continuing collaboration with the group of Professor Zoller at the University of Innsbruck has likewise been most important to our research activities.

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PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES